Alternate Mechanisms to the Toss in Cricket

Use Case Report

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Outline

In cricket, the toss of a coin is used to decide which team gets to decide the order in which the teams bat. While this might be good under some idealistic assumptions, data from past matches and analysis under slightly different assumptions shows that the toss might provide one team an advantage.

We seek to remedy this, and propose the use of a sealed-bid auction to decide which team gets to decide the order in which teams bat. We also argue that this mechanism is a suitable choice even when the teams do not have knowledge of their exact valuations by invoking the idea of Knightian auctions.

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1 Introduction

1.1 Motivation

A game of Test cricket is played between two teams of 11 players each over a period of 5 days. A game consists of four innings, and the teams alternate between batting and bowling in the four innings. Before the start of the game, the winner of a coin toss between the two teams gets to choose which team bats first. Given that the game is played over a long period of time, during which external conditions may change, the order in which the teams bat can have a big impact on the game, and therefore the winner of the coin toss is at an advantage of at the beginning of the game.

The idea of the toss has persisted under the assumption that, across many games played over the course of many years, the advantage gained by a team winning the toss will not matter, and the better team will win more often than not irrespective of the result of the toss. However, in retrospect, this assumption has not held up. As David Franklin wrote in Wisden [1], "[o]f the 1,397 matches that have produced a winner, 737 of those (53 per cent) were won by the team winning the toss, and 660 (47 per cent) by the team losing it." Given the number of matches under consideration, this difference is significant, especially in a highly competitive professional sport. Such scenarios become tricker for away teams as the home teams already have advantages of home conditions and might as well have altered the pitch to suit themselves.

In this work, we use mechanism design to study alternatives to the toss that try to negate the advantage within the game itself, and not across the games. We first analyse the toss as a mechanism, and consider the assumptions under which the toss works as intended in not giving the teams an *a priori* advantage in the game. We then explore alternative mechanisms to the toss, including sealed-bid auctions proposed David Franklin, and formally analyse the propoerties of these mechanisms under certain assumptions.

1.2 Problem definition

In our setting, we have two agents $N = \{\textsc{Team}_1, \textsc{Team}_2\}$ corresponding to the teams playing the match. Each agent has the same type set $\Theta_i = \{\theta^i_{\textsc{Bat}}, \theta^i_{\textsc{Bowl}}\}$, corresponding to the preference of a team to bat first, or bowl first. We define the set of outcomes as

$$X = \{(k, t_{\textsc{Team}_1}, t_{\textsc{Team}_2}) | k \in \{1, 2\}, t_{\textsc{Team}_1}, t_{\textsc{Team}_2} \in \mathbb{Z}\}$$

where k is the team that gets to bat first and t_i refers to the initial score, which is positive for the team that bats first and negative for the other team.

We assume a quasilinear environment, where each team's utility is defined as

$$u_i(x, \theta_i) = v_i(k, \theta_i) + t_i$$

where $x \in X$ and k is the team that wins the choice to decide the outcome. x.

In this setting, we seek to design a direct mechanism $\mathcal{D} = (\Theta_{\text{Team}_1}, \Theta_{\text{Team}_2}, f)$ with the social choice function f to result in an interesting and evenly contested cricket match.

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Existing coin toss mechanism

The existing coin toss mechanism could be modeled as a randomized mechanism [4] uniform over two deterministic mechanisms. For each outcome of the coin toss, we select a deterministic mechanism. Both the players don't pay anything after the coin toss. So the value of $t_i's$ would be 0 and $u_i(x, \theta_i) = v_i(k, \theta_i)$ for both the players. The existing social choice function chooses the team that bats first by the toss of a coin. In formal terms, the function is defined as follows for a type $\theta = \{\theta_H, \theta_A\}$

$$f(\theta) = \begin{cases} (\text{Team}_1, 0, 0) \text{ if Coin} = \text{Head} \\ (\text{Team}_2, 0, 0) \text{ if Coin} = \text{Tail} \end{cases}$$

where the Coin tossed is unbaised with $Pr[\text{Coin} = \text{Head}] = Pr[\text{Coin} = \text{Tail}] = 0.5$.

We analyze this mechanism under two different settings:

1. The two teams evaluate the pitch similarly. i.e. both either value batting first or bowling first. This is an idealistic assumption, and often teams prefer specific outcomes under certain conditions.

2. The two teams differ in their preferences and/or valuations. This is a more realistic setting that considers how teams may have specific preferences in certain conditions.

Case 1: Both teams have same preference

Consider a scenario where a test match is played on a dusty pitch. Both teams would like to bat first as the pitch would lose its shape and would become difficult to bat on as the game progresses. Batting first on day 1 earns Team_1 and Team_2 team some latent runs say r_1 and r_2 respectively. Since both teams would like to bat first, we neglect their beliefs θ^i_{Bowl} for bowling first on the pitch. So, the type set $\Theta_i = \theta^i_{\text{Bat}}$. The utility function $u_i : X \times \theta_i$ is defined for both the teams as:

$$u_1(\text{Team}_1, \theta^1_{\text{Bat}}) = r_1 \qquad u_2(\text{Team}_1, \theta^2_{\text{Bat}}) = 0$$
$$u_1(\text{Team}_2, \theta^1_{\text{Bat}}) = 0 \qquad u_2(\text{Team}_2, \theta^2_{\text{Bat}}) = r_2$$

We could also think of an exact opposite scenario where a test match is played on a wet pitch with overcast conditions. Both teams would like to bowl first as the conditions would aide the faster bowlers with the new ball. Batting first on day 1 would cost Team_1 and Team_2 team some latent runs say $-r_1$ and $-r_2$ respectively. Similar to the previous case, we neglect the players belief of batting first θ^i_{Bat}. So, the type set $\Theta_i = \theta^i_{\text{Bat}}$. The utility function $u_i : X \times \theta_i$ is defined for both the teams as:

$$u_1(\text{Team}_1, \theta^1_{\text{Bat}}) = -r_1 \qquad u_2(\text{Team}_1, \theta^2_{\text{Bat}}) = 0$$
$$u_1(\text{Team}_2, \theta^1_{\text{Bat}}) = 0 \qquad u_2(\text{Team}_2, \theta^2_{\text{Bat}}) = -r_2$$

Lemma 2.0.1. *Case 1 is optimal i.e no team has any advantage iff $r_1 = r_2$.*

Proof. To show that no team has any advantage is coin toss mechanism if $r_1 = r_2$ in case 1, it suffices to show that the mechanism is AE(allocatively efficient). This would imply that the resources(runs) are fairly distributed between both the teams. Since coin toss mechanism is a randomized mechanism, we need to prove that it is AE in expectation.

Consider the scenario where both teams want to bat first. Total expected valuation($\mathbb{E}[v_i(k, \theta_i)]$) for all $k \in K$:

$$\sum_{i \in N} \mathbb{E}[v_i(k, \theta_i)] = 0.5 \cdot (v_H(k, \theta^1_{\text{Bat}}) + v_A(k, \theta^2_{\text{Bat}})) \tag{1}$$

Using equation 1 we get:

$$k = \text{Team}_1 \qquad \sum_{i \in N} \mathbb{E}[v_i(\text{Team}_1, \theta_i)] = r_1/2$$
$$k = \text{Team}_2 \qquad \sum_{i \in N} \mathbb{E}[v_i(\text{Team}_2, \theta_i)] = r_2/2$$

Since $r_1 = r_2$, so the value of $\sum_{i \in N} \mathbb{E}[v_i(\text{Team}_1, \theta_i)] = \sum_{i \in N} \mathbb{E}[v_i(\text{Team}_2, \theta_i)]$. Similary, in the scenario when both teams want to bowl first, we can also show that $\sum_{i \in N} \mathbb{E}[v_i(\text{Team}_1, \theta_i)] = -r_1/2 = -r_2/2 = \sum_{i \in N} \mathbb{E}[v_i(\text{Team}_2, \theta_i)]$. Hence the mechanism is AE in expectation.

To prove that the converse is true, we observe that if no team has an advantage in the coin toss mechanism, it must be AE in expectation $\implies \mathbb{E}[v_H(., \theta_H)] = \mathbb{E}[v_A(., \theta_A)]$. For the first sceanrio, we have:

$$\sum_{i \in N} \mathbb{E}[v_i(\text{Team}_1, \theta_i)] = \sum_{i \in N} \mathbb{E}[v_i(\text{Team}_2, \theta_i)]$$
$$\implies r_1/2 = r_2/2$$
$$\implies r_1 = r_2$$

Similarly for the second scenario we have:

$$\sum_{i \in N} \mathbb{E}[v_i(\text{Team}_1, \theta_i)] = \sum_{i \in N} \mathbb{E}[v_i(\text{Team}_2, \theta_i)]$$
$$\implies -r_1/2 = -r_2/2$$
$$\implies r_1 = r_2$$

□

From lemma 2.0.1 we learn that when both teams have similar preferences, the existing coin toss mechanism is fair only under the assumption that both teams earn the same latent runs when they bat first. This seems to be an impractical assumption in a real world scenario because every team has its strengths and weakness. Considering various factors, the probability that two teams perform identically on the same pitch is almost negligible. So, the mechanism won't be allocatively efficient in expectation in most situations. Hence, in the real world, we can say a coin toss mechanism is not fair(optimal) when both teams have similar preferences and favor a team with an advantage in terms of latent runs.

Case 2: Both teams have different preference

Consider a scenario where a test match is played on a pitch that looks hard, and teams believe that it doesn't deteriorate as the game progresses. A team with a good bowling attack might prefer batting first because they feel that they can bowl out their opponents quickly in 4^{th} innings. On the other hand, a team with a solid batting lineup might prefer bowling first because they feel they can chase easily in the 4^{th} innings. In such a scenario, teams prefer a particular outcome but don't completely rule out the other outcome(not biased), unlike in case 2 where the team is certain of an outcome(biased) and we could neglect the belief of the other outcome. So, the type set for each player $\Theta_i = \{\theta^i_{\text{BAT}}, \theta^i_{\text{BOWL}}\}$. The team batting first could earn some runs and as well as also lose some runs based on its preferences. Let r_1 and r_2 be the net amount of runs earned by $\textsc{Team}_1$ and $\textsc{Team}_2$ respectively. Then the utility function $u_i : X \times \theta_i$ is defined for both the teams as:

$$u_1(\textsc{Team}_1, \theta_H) = r_1 \qquad u_2(\textsc{Team}_1, \theta_A) = 0$$
$$u_1(\textsc{Team}_2, \theta_H) = 0 \qquad u_2(\textsc{Team}_2, \theta_A) = r_2$$

where $\theta_H \in \Theta_H$ and $\theta_A \in \Theta_A$.

Lemma 2.0.2. *Case 2 is optimal i.e no team has any advantage iff $r_1 = r_2$.*

Proof. The proof is quite similar to the previous proof in lemma 2.0.1. We need to show that if if no team has advantage in case 2 when $r_1 = r_2$ by proving that the mechanism is AE(allocatively efficient). Total expected valuation($\mathbb{E}[v_i(k, \theta_i)]$) for every $k \in K$ is:

$$\sum_{i \in N} \mathbb{E}[v_i(k, \theta_i)] = 0.5 \sum_{\theta_H \in \Theta_H} v_H(k, \theta_H) + 0.5 \sum_{\theta_A \in \Theta_A} v_A(k, \theta_H) \tag{2}$$

Using equation 2 we get,

$$k = \textsc{Team}_1, \qquad \sum_{i \in N} \mathbb{E}[v_i(\textsc{Team}_1, \theta_i)] = r_1$$
$$k = \textsc{Team}_2, \qquad \sum_{i \in N} \mathbb{E}[v_i(\textsc{Team}_2, \theta_i)] = r_2$$

Since $r_1 = r_2$, so the value of $\sum_{i \in N} \mathbb{E}[v_i(\textsc{Team}_1, \theta_i)] = \sum_{i \in N} \mathbb{E}[v_i(\textsc{Team}_2, \theta_i)]$. Hence the mechanism is AE in expectation. Similar to previous proof, to prove converse is true, we say that the mechanism is AE. Using this we get:-

$$\sum_{i \in N} \mathbb{E}[v_i(\textsc{Team}_1, \theta_i)] = \sum_{i \in N} \mathbb{E}[v_i(\textsc{Team}_2, \theta_i)]$$
$$\implies r_1 = r_2$$

□

Similar to the previous case argument, we could say that in most scenarios, the mechanism won't be allocatively efficient because $r_1 = r_2$ is not a very realistic assumption. Hence, the coin toss mechanism is not fair(optimal) even when teams have different preferences and favors a team with an advantage of latent runs. To sum up, we formally proved that the coin toss mechanism is not fair(optimal) using lemma 2.0.1 and 2.0.2 except in some highly unlikely cases where both the teams perform identically same on the pitch.

3

Sealed bid auctions

The proposal of David Franklin is a direct mechanism similar to SCF1. Instead of the players making bids on an object with money, the players instead make bids on the choice to bat or field first with runs. Thus the value of $t_i's$ would be the runs each team is willing to bid and the choice to bat or field first is allocated to the winning bid. The payment is made in the form of runs to the losing bidder. Here we use the idea of negative runs for the analysis. However, in practice the losing bidder simply starts with extra runs as referenced in Fig. 1.

Let $f(\theta) = \{(k, t_1, t_2) | k \in \{1, 2\}, t_{\textsc{Team}_1}, t_{\textsc{Team}_2} \in \mathbb{Z}\}$.

3.1 David Franklin's proposal

David Franklin's proposal allocates the choice to the highest bidder at the highest price. More formally:

$$k(\theta) = \begin{cases} 1 \text{ if } \theta_1 \geq \theta_2 \\ 2 \text{ otherwise} \end{cases}.$$

Suppose we have an indicator function

$$I[k = i] = \begin{cases} 1 \text{ if } k = i \\ 0 \text{ otherwise} \end{cases}$$

Then the payment for the home team is

$$t_1(\theta) = -I[k = 1] \cdot \theta_1 + I[k = 2] \cdot \theta_2$$

.

Similarly the payment for the away team is

$$t_2(\theta) = -I[k = 2] \cdot \theta_2 + I[k = 1] \cdot \theta_1$$

.

Clearly this satisfies Strict Budget Balance (SBB) since the sum of payments is zero. The following lemmas explore the other properties of this function.

Lemma 3.1.1. *f is allocatively efficient.*

Proof. If $\theta_1 \geq \theta_2$, $v_1(k(\theta) = 1, \theta_1) + 0 = \theta_1 \geq v_2(k = 2, \theta_2) = \theta_2$. Similarly, if $\theta_1 < \theta_2$, $0 + v_2(k(\theta) = 2, \theta_2) = \theta_2 \geq v_1(k = 1, \theta_1) = \theta_1$. Hence f is AE.

□

Lemma 3.1.2. *The proposal is Ex-Post Efficient (EPE).*

Proof. Since f is both AE and SBB, by Lemma 18.2 in [3], f must be EPE. $\square$

Lemma 3.1.3. *f is not dominant-strategy incentive-compatible (DSIC).*

Proof. Suppose $\theta_1 \geq \theta_2$. Suppose 1 bids $\theta_2 \leq \hat{\theta} < \theta_1$. Then the utility for 1 is $u_1 = \theta_1 - \hat{\theta} > 0$. $\square$

Lemma 3.1.4. *f is non dictatorial.*

Proof. Since the utility function is quasi-linear, by Lemma 18.1 in [3], f cannot be a dictator. $\square$

3.2 A Vickrey auction-inspired proposal

Inspired by the idea of the highest bidder paying the second highest bid in the case of a Vickrey auction, here we consider the following mechanism.

$$k(\theta) = \begin{cases} 1 \text{ if } \theta_1 \geq \theta_2 \\ 2 \text{ otherwise} \end{cases} .$$

Suppose we have an indicator function

$$I[k = i] = \begin{cases} 1 \text{ if } k = i \\ 0 \text{ otherwise} \end{cases}$$

Then the payment for the home team is

$$t_1(\theta) = I[k = 1] \cdot \theta_1 - I[k = 2] \cdot \theta_2$$

.

Similarly the payment for the away team is

$$t_2(\theta) = I[k = 2] \cdot \theta_2 - I[k = 1] \cdot \theta_1$$

.

Again, this satisfies Strict Budget Balance (SBB) since the sum of payments is zero. The following lemmas explore the other properties of this function.

Lemma 3.2.1. *f is non dictatorial.*

Proof. Since the utility function is quasi-linear, by Lemma 18.1 in [3], f cannot be a dictator. $\square$

Lemma 3.2.2. *f is allocatively efficient.*

Proof. If $\theta_1 \geq \theta_2$, $v_1(k(\theta) = 1, \theta_1) + 0 = \theta_1 \geq v_2(k = 2, \theta_2) = \theta_2$. Similarly, if $\theta_1 < \theta_2$, $0 + v_2(k(\theta) = 2, \theta_2) = \theta_2 \geq v_1(k = 1, \theta_1) = \theta_1$. Hence f is AE. $\square$

Lemma 3.2.3. *The proposal is Ex-Post Efficient (EPE).*

Proof. Since f is both AE and SBB, by Lemma 18.2 in [3], f must be EPE. $\square$

Lemma 3.2.4. *f is dominant-strategy incentive-compatible (DSIC).*

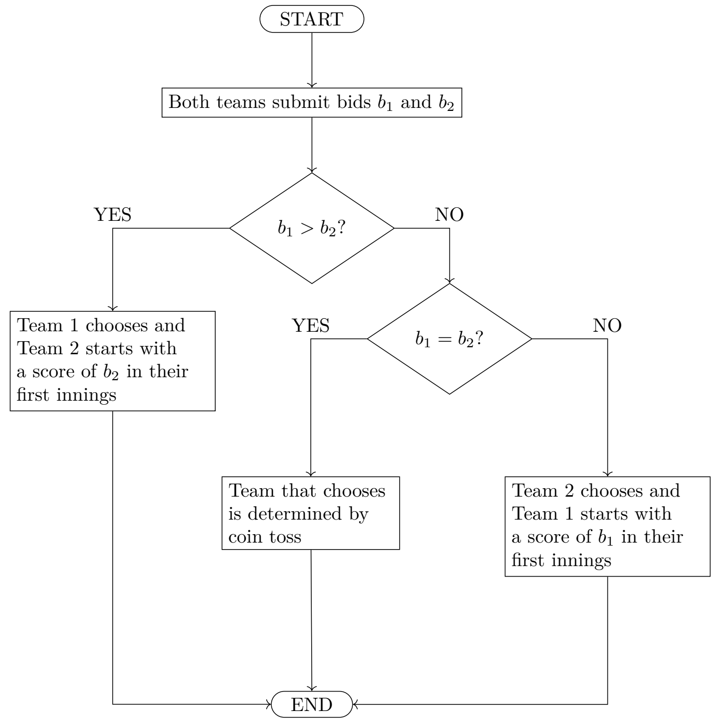

Figure 1: Flowchart depicting the second-prize sealed-bid auction mecahanism we propose, where one team gets to choose who bats in the first innings of the match in exchange for the other team being allotted a number of runs at the beginning of their first batting innings.

Proof. We'll show that this follows Groves' payment rule. Then this and the fact that f is AE implies that f is DSIC. In Groves' payment rule, $t_1 = v_2(k^*(\theta), \theta_2) + h_1(\theta_2)$, and $t_2 = v_1(k^*(\theta), \theta_1) + h_2(\theta_1)$. In the scheme,

$$t_1(\theta) = I[k = 1] \cdot \theta_1 - I[k = 2] \cdot \theta_2$$

. Since $v_1(k, \theta_1) = I[k = 1] \cdot \theta_1$, this can be re-written as

$$t_1(\theta) = v_1(k, \theta_1) + h_1(\theta_2)$$

where $h_1(\theta_2) = -I[k = 2] \cdot \theta_2$. Similarly, the same follows for $t_2(\theta)$. $\square$

The second-price auction is a mechanism that is deterministic, and hence simple to practically implement in the context of a cricket match. Figure 1 summarises the mechanism in a way that can be easily explained to players and viewers of the game. Additionally, under the assumption that players know their valuations exactly, the mechanism is DSIC, which is highly desirable.

4

Beyond exact valuations

So far, we have presented our arguments under the assumption that each team knows their valuation exactly. However, given the complexity of the real world situation, and the number of variables involved, this assumption can prove to be quite realistic.

We can relax this assumption, and instead consider the case when we assume both teams draw their valuations independently from the same distribution. However, even that assumption is quite strong given the nature of the game. Different teams have different strengths, and would thus value the pitch according to their strengths. For example, on the first day of a Test match in overcast conditions and a suitable pitch, a team with a strong fast bowling attack might choose to bowl first, while a team with a strong spin attack might choose to bat first in the hope of taking advantage of spin-friendly conditions in the final innings of the match.

We seek to understand how to design a mechanism when teams need not be certain of their exact valuations, and still try to obtain guarantees on the performance of the proposed mechanism. To do so, we turn to the concept of a Knightian auction proposed by [2].

In a Knightian auction, a player's valuation isn't guaranteed to be drawn from a known distribution. Instead, players are assumed to have only a set theoretic knowledge of their valuations. That is, the players only have the knowledge that their true valuation lies in a set of integers. They consider a case where the set of possible values is defined using a multiplicative inaccuracy around a chosen point. Given a number x, the set of possible valuations K_i for a player i is defined as $[(1-\delta)x, (1+\delta)x] \cap \mathbb{Z}$, where δ is the uncertainty a player has about their valuation. For example, if a team values the choice of batting first as somewhere between 45 and 55 runs, we could model this with $K = \{45, \ldots, 55\}$, $x = 50$, and $\delta = 0.1$ indicating a 10% uncertainty around the point 50. Note that while we define the set in terms of x, x is not related to the true value, and the true valuation maybe any element of the set K, and is unknown to the player i.

4.1 Definitions

The Knightian context for our cricket case is defined by

- Set of players $N = \{\textsc{Team}_1, \textsc{Team}_2\}$
- Set of valuations $\{0, 1, \ldots, B\}$ where B is the *valuation bound*. This is a reasonable assumption in cricket (given the finite amount of time), and the setting allows for the choice of a generous bound like $B = 100$.
- The uncertainty δ of the context, which is the maximum uncertainty among all the players
- The profile of candidate-valuation sets $K = (K_1, K_2)$ which are known to the players, where $K_i = [(1-\delta)x_i, (1+\delta)x_i] \cap \{0, 1, \ldots, B\}$ for some $x_i \in \mathbb{R}$
- The profile $\theta = (\theta_1, \theta_2)$ of true valuations of the players, which are unknown to the players themselves
- The profile X of outcomes, which are of the form (k, t_1, t_2) for allocation k, and run payments t_1, t_2
- The profile $u = (u_1, u_2)$ of utility functions, where

$$u_i(x) = v_i(k, \theta_i) + t_i$$

We also define the *maximum social welfare* as the sum of the value functions under the true valuations, $v_1(k, \theta_1) + v_2(k, \theta_2)$. Since v_i depends on the allocation k, one of these terms will be 0.

4.2 Dominant strategies

The first possibility considered by [2] is the existence of very weakly dominant strategy equilibria in Knightian auctions. This approach is directly extends the revelation theorem from exact valuations to candidate-valuation sets. For example, if we have a situation where $K_1 = \{45, \ldots, 55\}$ and $K_2 = \{55, \ldots, 65\}$, then for any pair of true valuations of the teams, the away team will have a higher valuation. In this case, K_2 dominates K_1.

[2] present formal definitions of dominance in the context of Knightian auctions in terms of mixed strategies of players over their candidate-valuation sets. For dominant strategy mechanisms, [2] prove a negative result. We state Theorem 1 from [2] informally here, and refer the reader to the original paper for the full statement and proof.

Theorem 4.1. *For all $n \geq 1$, $\delta \in (0, 1)$, and $B > \frac{3-\delta}{2\delta}$, no (possibly probabilistic) very-weakly-dominant-strategy-truthful mechanism can guarantee a fraction of the maximum social welfare greater than*

$$\frac{1}{n} + \frac{\lfloor \frac{3-\delta}{2\delta} \rfloor + 1}{B}$$

in any Knightian auction with n players, valuation bound B, and inaccuracy parameter δ.

Theorem 4.1 shows that the fraction of the maximum social welfare that a dominant strategy mechanism can never guarantee a fraction of the maximum social welfare that is inversely proportional to the number of players. In other words, any dominant strategy mechanism where players report their valuation ranges is unlikely to do better than selecting a winner uniformly at random. Since the toss already achieves this outcome, introducing a mechanism that adds more complexity without guaranteeing better outcomes is not desirable.

4.3 Second-price Knightian auctions

In contrast to the exact valuation setting, the second-price auction is not a dominant strategy incentive compatible mechanism in the Knightian setting. This is because each player does not know their true valuation anymore, and therefore cannot determine whether it is optimal to bid their true valuation. However, [2] still find that the second-price auction is guaranteed to achieve a lower-bounded fraction of the maximum social welfare.

We restate Theorem 2 from [2] informally, and direct the reader to the paper for the formal statement and proof.

Theorem 4.2. *In any Knightian auction with n players, valuation bound B, and inaccuracy parameter δ, the second-price mechanism (with ties broken randomly) guarantees a fraction of the maximum social welfare that is*

$$\left(\frac{1-\delta}{1+\delta}\right)^2$$

We illustrate this with an example. Consider Team 1 with candidate-valuation set $\{36, \ldots, 44\}$ and true value 42. Consider Team 2 with candidate-valuation set $\{41, \ldots, 49\}$ and true value 41. Both teams have an uncertainty of $\delta = 0.1$ in this case, and the maximum social welfare is 42. However, Team 1 (being conservative) bids 36, and Team 2 (being very eager) bids 49. Team 2 wins the bid, and the payment is 36 runs. Here, the outcome achieves $\frac{36}{42} \approx 0.86$ of the maximum social welfare, illustrating how this scheme achieves good outcomes even under uncertain circumstances.

In the limit of $\delta \to 0$, this converges to the Vickrey auction proposed in the case of teams knowing their exact valuations. Thus, in the second-price mechanism, we have a mechanism that has mathematical guarantees under strong assumptions of variability of the situation, and provides reasonable approximations to the maximum social welfare (even $\delta = 0.15$ allows teams to choose from ranges of 10 runs or more for bids around 50 runs while guaranteeing better outcomes than the toss).

A very important feature of this mechanism is that the designer need not be aware of δ. The value of δ is useful in deriving bounds, but the auction can proceed as usual. In fact, [2] also prove that among deterministic mechanisms, the second-price auction is optimal (see Theorem 3 of [2]).

4.4 Doing better than the second-price auction

[2] also presents a way to design a probabilistic undominated-strategy mechanism that achieves a better lower bound on the fraction of maximum social welfare. However, such probabilistic mechanism might not seem intuitive to players and audiences. In addition, the design of the mechanism requires knowledge of δ, which the designer cannot know exactly, and can vary from match to match. Thus, while it is possible in theory to do better than the second-price auction in the Knightian setting, such a mechanism might not be feasible to implement in the setting of a cricket match. We thus believe the second-price auction achieves a desirable balance between guarantees on optimality and ease of implementation.

Conclusion

In this report, we study mechanisms for choosing the order in which teams bat in a cricket match. We find that the existing coin toss mechanism has given a slight advantage to teams winning the toss, and relies on strong assumptions. We take the opportunity to explore alternate mechanisms, and show that when we assume teams know their exact valuations, a second-price auction is optimal. We also relax this assumption, and use the idea of Knightian auctions [2] to consider how the second-price auction performs when teams don't know their exact valuation. Given that the Knightian setting does not assume that the players know their valuation or even the distribution of their valuation, but instead choose a valuation from a range of values, we believe this analysis is better suited to studying the mechanism in a real world setting. Even in a Knightian auction, the second-price mechanism is shown to have usable guarantees. We therefore believe this would be a good mechanism to adopt for use in a cricket match.

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